EEL 316 Minor Test I Semester II 2014-2015

Answer *all questions* (Q.1: 20 marks, Q.2: 20 marks) Full Marks: 40

1. The transfer function $H(f)$ of a filter (with impulse response $h(t)$) that is matched to a pulse $g(t)$ having area 1 and support $[0,T]$ is given by

$$|H(f)| = A\,\text{sinc}^2\left(\frac{fT}{3}\right), \quad \text{angle}(H(f)) = -\frac{2\pi fT}{3}, \quad A, T > 0.$$

(a) Find $g(t)$ and sketch its plot, labeling the relevant portions. [10]

(b) Find T_{rms}, the r.m.s. duration of $g(t)$, given by [10]

$$T_{rms} = \left[\frac{\int_0^T t^2|g(t)|^2 dt}{\int_0^T |g(t)|^2 dt}\right]^{1/2}.$$

2. Consider the case of binary signaling over an AWGN channel in a bit interval $[0,T]$ with waveforms $s_1(t)$ (for symbol '1') and $s_0(t)$ (for symbol '0'), where the received signal is given by

$$x(t) = \begin{cases} s_0(t) + w(t) & \text{if symbol '0' is transmitted,} \\ s_1(t) + w(t) & \text{if symbol '1' is transmitted,} \end{cases}$$

$0 \le t \le T$, where $s_0(t)$ and $s_1(t)$ are given by

$$s_0(t) = \begin{cases} -\frac{2A_0 t}{T} & \text{for } 0 \le t \le \frac{T}{2}, \\ \frac{2A_0(t-T)}{T} & \text{for } \frac{T}{2} < t \le T, \\ 0 & \text{otherwise,} \end{cases} \quad s_1(t) = A_1\left[\text{rect}\left(\frac{t-(3T/4)}{T/2}\right) - \text{rect}\left(\frac{t-(T/4)}{T/2}\right)\right],$$

$0 < T/2$ $\quad$ $T/2$ $\quad$ T

where $0 < A_1 < A_0$. The additive noise $w(t)$ is a real-valued zero-mean white Gaussian random process with power spectral density $N_0/2$. The a priori probability of occurrence of symbol '0' is p_0 and that of symbol '1' is p_1. The receiver is implemented by passing $x(t)$ through a matched filter of impulse response $h(T-t)$ followed by sampling at $t = T$. Thus the receiver makes the decision $\int_0^T x(t)h(t)dt \underset{0}{\overset{1}{\gtrless}} \lambda$, and $h(t)$ satisfies the condition $\int_0^T s_1(t)h(t)dt > \int_0^T s_0(t)h(t)dt$. The threshold λ can be chosen to be either λ_{ML} or λ_{MAP}.

(a) Find $h(t)$ which minimizes the average symbol error probability (SEP), and satisfies $h(3T/4) = A_0 + 2A_1$, for an ML receiver as well as a MAP receiver, and sketch it, labeling the relevant portions. [6]

(b) If an ML receiver is used, and $h(t)$ is as in (a), find A_1/A_0 such that $\lambda_{ML} = 0$. Find the corresponding average SEP $P_{e,ML}$ in terms of A_1, T, and N_0. [6]

(c) If a MAP receiver is used with $h(t)$ is as in (a) and A_1/A_0 is as in (b), find λ_{MAP} and the corresponding average SEP $P_{e,MAP}$ in terms of A_1, T, N_0, and p_1/p_0. [4]

(d) For $A_1^2 T = 16N_0$ and $p_1 = \sqrt{e}/(1+\sqrt{e})$, calculate $P_{e,MAP}/P_{e,ML}$, where $P_{e,ML}$ is as in (b) and $P_{e,MAP}$ is as in (c). [4]
